練習問題で in.→ out.

[双曲線関数連続講義 #3] テーマ：基本の微積分

1min.

1 次を証明せよ．ただし、C は任意定数とする．

(1) $\displaystyle\int \frac{1}{\sinh^2 x}dx = -\frac{1}{\tanh x} + C$

proof

$$\left(-\frac{1}{\tanh x}\right)' = -\left(\frac{\cosh x}{\sinh x}\right)' = -\frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x} = \frac{1}{\sinh^2 x}$$

∎

(2) $\displaystyle\int \frac{1}{\sinh x}dx = \log\sqrt{\frac{\cosh x - 1}{\cosh x + 1}} + C$

$$\int \frac{1}{\sinh x}dx = \int \frac{\sinh x}{\sinh^2 x}dx = \int \frac{\sinh x}{(\cosh x+1)(\cosh x-1)}dx$$

$$= \frac{-1}{2}\int\left(\frac{\sinh x}{\cosh x+1} - \frac{\sinh x}{\cosh x-1}\right)dx$$

$$= -\frac{1}{2}\left(\log|\cosh x+1| - \log|\cosh x-1|\right) + C$$

$$= -\frac{1}{2}\log\left|\frac{\cosh x+1}{\cosh x-1}\right| + C = \log\sqrt{\frac{\cosh x-1}{\cosh x+1}} + C$$

∎

2 次を計算せよ．

(1) $\displaystyle\int \sinh 3x\, dx$

$$= \frac{1}{3}\cosh 3x + C \quad //$$

(2) $\displaystyle\int \sinh 3x \cosh 2x\, dx$

$$= \frac{1}{2}\int(\sinh 5x + \sinh x)dx$$

$$= \frac{1}{2}\left(\frac{1}{5}\cosh 5x + \cosh x\right) + C \quad //$$

(※ $\sinh(3x+2x) = \sinh 3x\cosh 2x + \cancel{\cosh 3x \sinh 2x}$

$+)\ \sinh(3x-2x) = \sinh 3x\cdot\cosh 2x - \cancel{\cosh 3x \sinh 2x}$

$\sinh 5x + \sinh x = 2\cdot\sinh 3x\cdot\cosh 2x$)

(3) $\displaystyle\int x\sinh 3x\, dx$

$$= \int x\cdot\left(\frac{1}{3}\cosh 3x\right)'dx$$

$$= \frac{1}{3}x\cdot\cosh 3x - \frac{1}{3}\int\cosh 3x\, dx$$

$$= \frac{1}{3}x\cdot\cosh 3x - \frac{1}{9}\sinh 3x + C \quad //$$

(4) $\displaystyle\int \sinh^3 x\cosh^2 x\, dx$

$$= \int \sinh x\cdot(\sinh x)^2\cdot\cosh^2 x\, dx$$

$$= \int \sinh x\cdot(\cosh^4 x - \cosh^2 x)dx$$

$$= \frac{1}{5}\cosh^5 x - \frac{1}{3}\cosh^3 x + C \quad //$$

3 α を正の定数とする．

(1)次を示せ．

$$\int_{-\alpha}^{\alpha} e^x\cos(\sinh x)\,dx = \int_{-\alpha}^{\alpha} e^{-x}\cos(\sinh x)\,dx$$

proof

$$\int_{-\alpha}^{\alpha} e^x\cdot\cos(\sinh x)dx$$

$dx = -dt$, $x = -t$

x	$-\alpha \to \alpha$
t	$\alpha \to -\alpha$

$$= \int_{-\alpha}^{\alpha} e^{-t}\cdot\cos(\sinh(-t))dt$$

$$= \int_{-\alpha}^{\alpha} e^{-t}\cdot\cos(-\sinh t)dt$$

$$= \int_{-\alpha}^{\alpha} e^{-x}\cos(\sinh x)dx$$

∎

(2) 次を計算せよ．

$$\int_{-\alpha}^{\alpha} e^x\cos(\sinh x)\,dx$$

(解)

$I = \displaystyle\int_{-\alpha}^{\alpha} e^x\cdot\cos(\sinh x)dx$ とおくと．

(1)

$$I = \frac{1}{2}\cdot\left(\int_{-\alpha}^{\alpha} e^x\cdot\cos(\sinh x)dx + \int_{-\alpha}^{\alpha} e^{-x}\cdot\cos(\sinh x)dx\right)$$

$$= \int_{-\alpha}^{\alpha}\frac{e^x+e^{-x}}{2}\cos(\sinh x)dx$$

$$= \int_{-\alpha}^{\alpha}\cosh x\cdot\cos(\sinh x)dx = \Big[\sin(\sinh x)\Big]_{-\alpha}^{\alpha}$$

$$= \sin(\sinh\alpha) - \sin(\sinh(-\alpha)) = 2\cdot\sin(\sinh\alpha) \quad //$$