

$$1. 4a^2b + 2ab^2 + ab \\ = ab(2a + b + 1)$$

$$2. a^2b + ab(b+1) \\ = ab \{ a + (b+1) \} \\ = ab(a + b + 1)$$

$$3. x^2 + 4x + 4 = (x+2)^2$$

$$4. x^2 + 18x + 81 = (x+9)^2$$

$$5. x^2 - 13x + 30 = (x-10)(x-3)$$

$$6. x^2 + 5x - 6 = (x+6)(x-1)$$

$$7. x^2 + 5x + 6 = (x+2)(x+3)$$

$$8. x^2 - 12x + 20 = (x-2)(x-10)$$

$$9. x^2 + 11x + 24 = (x+3)(x+8)$$

$$10. -18 - 3x + x^2 = x^2 - 3x - 18 \\ = (x-6)(x+3)$$

$$11. 4x^2 - 20x + 25 = (2x-5)^2$$

$$12. 3x^2 + 48x + 192 = (3x+16)(x+12)$$

$$\begin{array}{r} 3 \times 16 = 16 \\ 1 \times 12 = 36 \\ \hline 3 \quad 192 \quad 32 \end{array}$$

$$13. a^2b^2 - 4 = (ab+2)(ab-2)$$

$$14. 5x^2 + 13x + 6 = (x+2)(5x+3)$$

$$\begin{array}{r} 1 \times 2 = 10 \\ 5 \times 3 = 3 \\ \hline 5 \quad 6 \quad 13 \end{array}$$

$$15. 2x^2 + x - 6 = (x+2)(2x-3)$$

$$\begin{array}{r} 1 \times 2 = (+4) \\ 2 \times (-3) = (-3) \\ \hline 2 \quad -6 \quad 1 \end{array}$$

$$16. 5x^2 + 7x + 2 = (x+1)(5x+2)$$

$$\begin{array}{r} 1 \times 1 = 5 \\ 5 \times 2 = 2 \\ \hline 5 \quad 2 \quad 7 \end{array}$$

$$17. 3x^2 - 10x + 3 = (x-3)(3x-1)$$

$$\begin{array}{r} 1 \times (-3) = (-9) \\ 3 \times (-1) = (-3) \\ \hline 3 \quad 3 \quad -10 \end{array}$$

$$18. 4x^2 - 4x - 15 = (2x-5)(2x+3)$$

$$\begin{array}{r} 2 \times (-5) = (-10) \\ 2 \times 3 = 6 \\ \hline 4 \quad -15 \quad -4 \end{array}$$

$$19. 3x^2 + 11x + 10 = (3x+5)(x+2)$$

$$\begin{array}{r} 3 \times 5 = 15 \\ 1 \times 2 = 2 \\ \hline 3 \quad 10 \quad 11 \end{array}$$

$$20. 4x^2 - 23x - 27 = (4x-27)(x+1)$$

$$\begin{array}{r} 4 \times (-27) = (-108) \\ 1 \times 1 = 1 \\ \hline 4 \quad -27 \quad -23 \end{array}$$

$$21. 10x^2 - 21x - 10 = (2x-5)(5x+2)$$

$$\begin{array}{r} 2 \times (-5) = (-10) \\ 5 \times 2 = 10 \\ \hline 10 \quad -10 \quad -21 \end{array}$$

$$22. 5x^2 + 7x - 6 = (5x-3)(x+2)$$

$$\begin{array}{r} 5 \times (-3) = (-15) \\ 1 \times 2 = 2 \\ \hline 5 \quad -6 \quad 7 \end{array}$$

$$23. 36x^2 - 73x - 18 = (4x-9)(9x+2)$$

$$\begin{array}{r} 4 \times (-9) = (-36) \\ 9 \times 2 = 18 \\ \hline 36 \quad -18 \quad -73 \end{array}$$

$$24. 6x^2 - xy - 12y^2 = (2x-3y)(3x+4y)$$

$$\begin{array}{r} 2 \times (-3) = (-6) \\ 3 \times 4 = 12 \\ \hline 6 \quad -12 \quad -1 \end{array}$$

$$25. 12x^2 + 5x - 72 = (4x-9)(3x+8)$$

$$\begin{array}{r} 4 \times (-9) = (-36) \\ 3 \times 8 = 24 \\ \hline 12 \quad -36 \quad 5 \end{array}$$

$$26. x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$$27. 8x^3 + 8 = 8(x^3 + 1) \\ = 8(x+1)(x^2 - x + 1)$$

$$28. 27a^3 - 8b^3$$

$$= (3a - 2b)(9a^2 + 6ab + 4b^2)$$

$$29. a^3 - 6a^2b + 9ab^2$$

$$= a(a^2 - 6ab + 9b^2) = a(a - 3b)^2$$

$$30. x^4 + 8x = x(x^3 + 8)$$

$$= x(x+2)(x^2 - 2x + 4)$$

$$31. 2a - 3b + 3ab - 2$$

$$= 2a - 2 - 3b + 3ab$$

$$= 2(a-1) + 3b(a-1)$$

$$= (a-1)(2+3b)$$

$$32. x^2 - 6x - y^2 + 9$$

$$= x^2 - 6x - (y-3)(y+3)$$

$$= \{x + (y-3)\}\{x - (y+3)\}$$

$$= (x+y-3)(x-y-3)$$

$$33. 2x^2 - 18y^2$$

$$= 2(x^2 - 9y^2) = 2(x-3y)(x+3y)$$

$$34. a^2 - b^2 + 4bc - 4c^2$$

$$= a^2 - (b^2 - 4bc + 4c^2)$$

$$= a^2 - (b-2c)^2$$

$$= \{a - (b-2c)\}\{a + (b-2c)\}$$

$$= (a-b+2c)(a+b-2c)$$

$$35. a^2 + b^2 + 2ab + ac + bc$$

$$= (a+b)^2 + c(a+b)$$

$$= (a+b)\{a+b+c\}$$

$$= (a+b)(a+b+c)$$

$$36. x^2 - y^2 + x^3 - y^3 + x - y$$

$$= (x+y)(x-y) + (x-y)(x^2 + xy + y^2) + (x-y)$$

$$= (x-y)\{x+y + (x^2 + xy + y^2) + 1\}$$

$$= (x-y)(x^2 + xy + y^2 + x + y + 1)$$

$$37. x^4 - 8x^2 + 12 = (x^2 - 2)(x^2 - 6)$$

$$38. x^4 + x^2 - 2 = (x^2 + 2)(x^2 - 1)$$

$$= (x^2 + 2)(x+1)(x-1)$$

$$39. x^4 - 15x^2 + 9 = (x^2 - 3)^2 - 9x^2$$

$$= \{(x^2 - 3) + 3x\}\{(x^2 - 3) - 3x\}$$

$$= (x^2 + 3x - 3)(x^2 - 3x - 3)$$

$$40. x^4 - 1 = (x^2 + 1)(x^2 - 1)$$

$$= (x^2 + 1)(x+1)(x-1)$$

$$41. x^4 - 1 = (x^4 + 1)(x^4 - 1)$$

$$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$$

$$= (x^4 + 1)(x^2 + 1)(x+1)(x-1)$$

$$42. x^6 - 1 = (x^3 + 1)(x^3 - 1)$$

$$= (x+1)(x^2 - x + 1)(x-1)(x^2 + x + 1)$$

$$43. a^6 - 9a^3b^3 + 8b^6$$

$$= (a^3 - b^3)(a^3 - 8b^3)$$

$$= (a-b)(a^2 + ab + b^2)(a-2b)(a^2 + 2ab + 4b^2)$$

$$44. (x+2)^2 + 3(x+2) + 2$$

$$= \{(x+2) + 1\}\{(x+2) + 2\}$$

$$= (x+3)(x+4)$$

$$45. (x-y)^2 - x + y$$

$$= (x-y)^2 - (x-y)$$

$$= (x-y)\{(x-y) - 1\} = (x-y)(x-y-1)$$

$$46. 8(x+y)^2 + 6x + 6y + 1$$

$$= 8(x+y)^2 + 6(x+y) + 1$$

$$= \{2(x+y) + 1\}\{4(x+y) + 1\}$$

$$= (2x+2y+1)(4x+4y+1)$$

$$47. x^2 - y^2 - 6y - 9$$

$$= x^2 - (y^2 + 6y + 9)$$

$$= x^2 - (y+3)^2$$

$$= \{x - (y+3)\}\{x + (y+3)\}$$

$$= (x-y-3)(x+y+3)$$

$$\begin{aligned}
 48. \quad & x^2 + 2xy + y^2 - x - y - 2 \\
 &= (x+y)^2 - (x+y) - 2 \\
 &= \{(x+y) - 2\} \{(x+y) + 1\} \\
 &= (x+y-2)(x+y+1)
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & x^2 - 2xy + y^2 - 4z^2 \\
 &= (x-y)^2 - 4z^2 \\
 &= \{(x-y) - 2z\} \{(x-y) + 2z\} \\
 &= (x-y-2z)(x-y+2z)
 \end{aligned}$$

$$\begin{aligned}
 50. \quad & 5z^2 - 6z(x+y) + (x+y)^2 \\
 &= \{5z - (x+y)\} \{z - (x+y)\} \\
 &= (5z - x - y)(z - x - y)
 \end{aligned}$$

$$\begin{aligned}
 51. \quad & (x+3y)(x+3y-3z) + 2z^2 \\
 &= (x+3y)^2 - 3z(x+3y) + 2z^2 \\
 &= \{(x+3y) - 2z\} \{(x+3y) - z\} \\
 &= (x+3y-2z)(x+3y-z)
 \end{aligned}$$

$$\begin{aligned}
 52. \quad & (x+y+1)(x+y-2) - 4 \\
 &= (x+y)^2 - (x+y) - 2 - 4 \\
 &= (x+y)^2 - (x+y) - 6 \\
 &= \{(x+y) - 3\} \{(x+y) + 2\} \\
 &= (x+y-3)(x+y+2)
 \end{aligned}$$

$$\begin{aligned}
 53. \quad & (x+y+z)(x-3y+z) - 5y^2 \\
 &= (x+z+y)(x+z-3y) - 5y^2 \\
 &= (x+z)^2 - 2y(x+z) - 3y^2 - 5y^2 \\
 &= (x+z)^2 - 2y(x+z) - 8y^2 \\
 &= \{(x+z) - 4y\} \{(x+z) + 2y\} \\
 &= (x-4y+z)(x+2y+z)
 \end{aligned}$$

$$\begin{aligned}
 54. \quad & (x^2+x+1)(x^2+x+3) - 15 \\
 &= (x^2+x)^2 + 4(x^2+x) + 3 - 15 \\
 &= (x^2+x)^2 + 4(x^2+x) - 12 \\
 &= \{(x^2+x) + 6\} \{(x^2+x) - 2\} \\
 &= (x^2+x+6)(x+2)(x-1)
 \end{aligned}$$

$$\begin{aligned}
 55. \quad & x(x+1)(x+2)(x+3) - 8 \\
 &= x(x+3)(x+1)(x+2) - 8 \\
 &= (x^2+3x)(x^2+3x+2) - 8 \\
 &= (x^2+3x)^2 + 2(x^2+3x) - 8 \\
 &= \{(x^2+3x) + 4\} \{(x^2+3x) - 2\} \\
 &= (x^2+3x+4)(x^2+3x-2)
 \end{aligned}$$

$$\begin{aligned}
 56. \quad & (x-3)(x-1)(x-5)(x-7) + 15 \\
 &= (x-3)(x-5)(x-1)(x-7) + 15 \\
 &= (x^2-8x+15)(x^2-8x+7) + 15 \\
 &= (x^2-8x)^2 + 22(x^2-8x) + 105 + 15 \\
 &= (x^2-8x)^2 + 22(x^2-8x) + 120 \\
 &= \{(x^2-8x) + 12\} \{(x^2-8x) + 10\} \\
 &= (x-2)(x-6)(x^2-8x+10)
 \end{aligned}$$

$$\begin{aligned}
 57. \quad & (x^2-2x)^2 - 18(x^2-2x) + 80 \\
 &= \{(x^2-2x) - 9\} \{(x^2-2x) - 10\} \\
 &= (x-4)(x+2)(x^2-2x-10)
 \end{aligned}$$

$$\begin{aligned}
 58. \quad & 2ax^2 + (6-a^2)x - 3a \\
 &= (2x-a)(ax+3) \quad \begin{array}{r} 2 \times (-a) = -a^2 \\ a \times 3 = 6 \\ \hline 2a \quad -3a \quad 6-a^2 \end{array}
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & abx^2 - (a^2+b^2)xy + aby^2 \\
 & \begin{array}{r} a \times (-b) = -b^2 \\ b \times (-a) = -a^2 \\ \hline ab \quad ab \quad -(a^2+b^2) \end{array} \\
 &= (ax-by)(bx-ay)
 \end{aligned}$$

$$\begin{aligned}
 60. \quad & abx^2 + (a^2-b^2)xy - aby^2 \\
 & \begin{array}{r} a \times (-b) = -b^2 \\ b \times a = a^2 \\ \hline ab \quad -ab \quad a^2-b^2 \end{array} \\
 &= (ax-by)(bx+ay)
 \end{aligned}$$

$$\begin{aligned}
 61. \quad & xy - x - y + 1 \\
 &= x(y-1) - (y-1) \\
 &= (y-1)(x-1)
 \end{aligned}$$

$$\begin{aligned}
62. \quad & a^2b^2 - a^2 - b - 1 \\
& = a^2(b^2 - 1) - (b + 1) \\
& = a^2(b + 1)(b - 1) - (b + 1) \\
& = (b + 1) \{ a^2(b - 1) - 1 \} \\
& = (b + 1)(a^2b - a^2 - 1)
\end{aligned}$$

$$\begin{aligned}
63. \quad & 25 - 15y + 3xy - x^2 \\
& = -(x^2 - 3xy + 15y - 25) \\
& = -\{x^2 - 3xy + 5(3y - 5)\} \\
& = -[(x - 5)\{x - (3y - 5)\}] \\
& = -(x - 5)(x - 3y + 5)
\end{aligned}$$

$$\begin{aligned}
64. \quad & ab + bc - cd - da \\
& = b(a + c) - d(a + c) \\
& = (a + c)(b - d)
\end{aligned}$$

$$\begin{aligned}
65. \quad & a^2 - b^2 - ac - bc \\
& = (a + b)(a - b) - c(a + b) \\
& = (a + b)(a - b - c)
\end{aligned}$$

$$\begin{aligned}
66. \quad & a^2 + b^2 - c^2 - 2ab \\
& = (a - b)^2 - c^2 \\
& = (a - b + c)(a - b - c)
\end{aligned}$$

$$\begin{aligned}
67. \quad & 9a^2 - b^2 - 4bc - 4c^2 \\
& = 9a^2 - (b^2 + 4bc + 4c^2) \\
& = 9a^2 - (b + 2c)^2 \\
& = (3a + b + 2c)(3a - b - 2c)
\end{aligned}$$

$$\begin{aligned}
68. \quad & 4a^2 - c^2 - b^2 + 2bc \\
& = 4a^2 - (c^2 - 2bc + b^2) \\
& = 4a^2 - (b - c)^2 \\
& = (2a + b - c)(2a - b + c)
\end{aligned}$$

$$\begin{aligned}
69. \quad & a^2 + b^2 - 6c^2 + 2ab + bc + ca \\
& = (a + b)^2 + c(a + b) - 6c^2 \\
& = \{(a + b) + 3c\} \{(a + b) - 2c\} \\
& = (a + b + 3c)(a + b - 2c)
\end{aligned}$$

$$\begin{aligned}
70. \quad & a^2 - b^2 - c^2 + d^2 + 2ab + 2bc \\
& = a^2 + 2ad + d^2 - (b^2 - 2bc + c^2) \\
& = (a + d)^2 - (b - c)^2 \\
& = \{(a + d) + (b - c)\} \{(a + d) - (b - c)\} \\
& = (a + b - c + d)(a - b + c + d)
\end{aligned}$$

$$\begin{aligned}
71. \quad & x^2 + (2y + 3)x + (y + 1)(y + 2) \\
& = (x + y + 1)(x + y + 2)
\end{aligned}$$

$$\begin{aligned}
72. \quad & x^2 + xy - 2y^2 + 2x + 7y - 3 \\
& = x^2 + (y + 2)x - (2y^2 - 7y + 3) \\
& = x^2 + (y + 2)x - (2y - 1)(y - 3) \\
& = \{x + (2y - 1)\} \{x - (y - 3)\} \\
& = (x + 2y - 1)(x - y + 3)
\end{aligned}$$

$$\begin{aligned}
73. \quad & x^2 + 4xy + 3y^2 - 8x - 6y - 9 \\
& = x^2 + (4y - 8)x + 3(y^2 - 2y - 3) \\
& = x^2 + (4y - 8)x + 3(y - 3)(y + 1) \\
& = \{x + 3(y - 3)\} \{x + (y + 1)\} \\
& = (x + 3y - 9)(x + y + 1)
\end{aligned}$$

$$\begin{aligned}
74. \quad & x^2 + xy + 2x + y + 1 \\
& = x^2 + (y + 2)x + y + 1 \\
& = \{x + (y + 1)\} \{x + 1\} \\
& = (x + y + 1)(x + 1)
\end{aligned}$$

$$\begin{aligned}
75. \quad & x^2 - 2xy + y^2 - x + y - 2 \\
& = (x - y)^2 - (x - y) - 2 \\
& = \{(x - y) - 2\} \{(x - y) + 1\} \\
& = (x - y - 2)(x - y + 1)
\end{aligned}$$

$$\begin{aligned}
76. \quad & x^2 + 3xy + 2y^2 - 6x - 11y + 5 \\
& = x^2 + 3(y - 2)x + 2y^2 - 11y + 5 \\
& = x^2 + 3(y - 2)x + (2y - 1)(y - 5) \\
& = (x + 2y - 1)(x + y - 5)
\end{aligned}$$

$$\begin{aligned}
 77. \quad & x^2 + xy + 4x - 2y^2 + 5y + 3 \\
 & = x^2 + (y+4)x - (2y^2 - 5y - 3) \\
 & = x^2 + (y+4)x - (2y+1)(y-3) \\
 & = \{x + (2y+1)\} \{x - (y-3)\} \\
 & = (x+2y+1)(x-y+3)
 \end{aligned}$$

$$\begin{aligned}
 78. \quad & 2x^2 - 3xy - 2y^2 - 5x + 5y + 3 \\
 & = 2x^2 - (3y+5)x - (2y^2 - 5y - 3) \\
 & = 2x^2 - (3y+5)x - (2y+1)(y-3) \\
 & \quad \begin{array}{r} 2 \quad \times \quad (y-3) = y-3 \\ 1 \quad \times \quad -(2y+1) = -(4y+2) \\ \hline 2 \quad -(y-3)(2y+1) \quad -3y-5 \end{array} \\
 & = \{2x + (y-3)\} \{x - (2y+1)\} \\
 & = (2x+y-3)(x-2y-1)
 \end{aligned}$$

$$\begin{aligned}
 79. \quad & 2x^2 + 3xy + 2y^2 + 5x + y - 3 \\
 & = 2x^2 + 5(y+1)x + 2y^2 + y - 3 \\
 & = 2x^2 + 5(y+1)x + (2y+3)(y-1) \\
 & \quad \begin{array}{r} 2 \quad \times \quad (y-1) = y-1 \\ 1 \quad \times \quad (2y+3) = 4y+6 \\ \hline 2 \quad (y-1)(2y+3) \quad 5y+5 \end{array} \\
 & = (2x+y-1)(x+2y+3)
 \end{aligned}$$

$$\begin{aligned}
 80. \quad & 2x^2 + 2xy - 3x - 4y - 2 \\
 & = 2x^2 + (2y-3)x - 2(2y+1) \\
 & \quad \begin{array}{r} 2 \quad \times \quad (2y+1) = 2y+1 \\ 1 \quad \times \quad -2 = -4 \\ \hline 2 \quad -2(2y+1) \quad 2y-3 \end{array} \\
 & = (2x+2y+1)(x-2)
 \end{aligned}$$

$$\begin{aligned}
 81. \quad & 2ax^2 - (3a-2b)xy - 3by^2 \\
 & \quad - (5a-2c)x - (5b+3c)y - 5c \\
 & = 2ax^2 - \{(3a-2b)y + (5a-2c)\}x \\
 & \quad - \{3by^2 + (5b+3c)y + 5c\} \\
 & = 2ax^2 - \{(3a-2b)y + (5a-2c)\}x \\
 & \quad - (3y+5)(by+c) \\
 & = \{2x - (3y+5)\} \{ax - (by+c)\} \\
 & = (2x-3y-5)(ax-by-c)
 \end{aligned}$$

$$\begin{aligned}
 82. \quad & x^3 - 6x^2 + (2x-8) \\
 & = x^3 - 8 - 6x(x-2) \\
 & = (x-2)(x^2+2x+4) - 6x(x-2) \\
 & = (x-2)(x^2+2x+4-6x) \\
 & = (x-2)(x^2-4x+4) \\
 & = (x-2)(x-2)^2 = (x-2)^3
 \end{aligned}$$

$$\begin{aligned}
 83. \quad & x^3 - 5x^2 - 4x + 20 \\
 & = x^2(x-5) - 4(x-5) \\
 & = (x-5)(x^2-4) \\
 & = (x-5)(x-2)(x+2)
 \end{aligned}$$

$$\begin{aligned}
 84. \quad & x^2y + 4y^2z - 4y^3 - x^2z \\
 & = x^2(y-z) - 4y^2(y-z) \\
 & = (y-z)(x^2-4y^2) \\
 & = (y-z)(x+2y)(x-2y)
 \end{aligned}$$

$$\begin{aligned}
 85. \quad & a^k + a^2c - ab^2 + abc + b^2c \\
 & = (a^k - ab^2) + a^2c + abc + b^2c \\
 & = a(a^3 - b^3) + c(a^2 + ab + b^2) \\
 & = a(a-b)(a^2 + ab + b^2) + c(a^2 + ab + b^2) \\
 & = (a^2 + ab + b^2) \{a(a-b) + c\} \\
 & = (a^2 + ab + b^2)(a^2 - ab + c)
 \end{aligned}$$

$$\begin{aligned}
 86. \quad & x^8 - 3x^2 + 9 \\
 & = (x^2+3)^2 - 9x^2 \\
 & = (x^2+3x+3)(x^2-3x+3)
 \end{aligned}$$

$$\begin{aligned}
 87. \quad & x^4 - 6x^2 + 1 \\
 & = (x^2-1)^2 - 4x^2 \\
 & = (x^2+2x-1)(x^2-2x-1)
 \end{aligned}$$

$$\begin{aligned}
 88. \quad & x^8 + x^7y^4 + y^8 \\
 & = (x^4 + y^4)^2 - x^7y^4 \\
 & = (x^4 + x^2y^2 + y^4)(x^4 - x^2y^2 + y^4)
 \end{aligned}$$

$$\begin{aligned}
 89. \quad & x^4 + 4 \\
 & = (x^2+2)^2 - 4x^2 \\
 & = (x^2+2x+2)(x^2-2x+2)
 \end{aligned}$$

$$\begin{aligned}
 90. \quad & x^5 + x^4 + x^3 + x^2 + x + 1 \\
 &= x^3(x^2 + x + 1) + x^2 + x + 1 \\
 &= (x^2 + x + 1)(x^3 + 1) \\
 &= (x^2 + x + 1)(x^2 - x + 1)(x + 1)
 \end{aligned}$$

$$\begin{aligned}
 91. \quad & a^2(b+c) + b^2(c+a) + c^2(a+b) + 3abc \\
 &= (b+c)a^2 + (b^2 + c^2 + 3bc)a + b^2c + bc^2 \\
 &= (b+c)a^2 + (b^2 + 3bc + c^2)a + bc(b+c) \\
 &\quad \begin{array}{r} (b+c) \quad bc = bc \\ 1 \quad \times \quad (b+c) = b^2 + 2bc + c^2 \\ \hline (b+c) \quad bc(b+c) \quad b^2 + 3bc + c^2 \end{array} \\
 &= \{ (b+c)a + bc \} \{ a + (b+c) \} \\
 &= (a+b+c)(ab+bc+ca)
 \end{aligned}$$

$$\begin{aligned}
 92. \quad & a^2(b-c) + b^2(c-a) + c^2(a-b) \\
 &= (b-c)a^2 + (c^2 - b^2)a + b^2c - bc^2 \\
 &= (b-c)a^2 - (b^2 - c^2)a + bc(b-c) \\
 &= (b-c)a^2 - (b-c)(b+c)a + bc(b-c) \\
 &= (b-c) \{ a^2 - (b+c)a + bc \} \\
 &= (b-c)(a-b)(a-c) \\
 &= -(a-b)(b-c)(c-a)
 \end{aligned}$$

$$\begin{aligned}
 93. \quad & (ab+1)(a+1)(b+1) + ab \\
 &= (ab+1)(ab+a+b+1) + ab \\
 &= (ab+1)^2 + (a+b)(ab+1) + ab \\
 &= \{ (ab+1) + a \} \{ (ab+1) + b \} \\
 &= (ab+a+1)(ab+b+1)
 \end{aligned}$$

$$\begin{aligned}
 94. \quad & (a^2-1)(b^2-1) - 4ab \\
 &= a^2b^2 - a^2 - b^2 + 1 - 4ab \\
 &= (b^2-1)a^2 - 4b \cdot a - (b^2-1) \\
 &= (b+1)(b-1)a^2 - 4ba - (b+1)(b-1) \\
 &\quad \begin{array}{r} (b+1) \quad (b-1) = b^2 - 2b + 1 \\ (b-1) \quad \times \quad -(b+1) = -(b^2 + 2b + 1) \\ \hline (b+1)(b-1) \quad -(b+1)(b-1) \quad -4b \end{array} \\
 &= \{ (b+1)a + (b-1) \} \{ (b-1)a - (b+1) \} \\
 &= (ab+a+b-1)(ab-a-b-1)
 \end{aligned}$$

$$\begin{aligned}
 95. \quad & a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc \\
 &= (b+c)a^2 + (b^2 + c^2 + 2bc)a + bc(b+c) \\
 &= (b+c)a^2 + (b+c)^2a + bc(b+c) \\
 &= (b+c) \{ a^2 + (b+c)a + bc \} \\
 &= (b+c)(a+b)(a+c) \\
 &= (a+b)(b+c)(c+a)
 \end{aligned}$$

$$\begin{aligned}
 96. \quad & a(b^2-c^2) + b(c^2-a^2) + c(a^2-b^2) \\
 &= (c-b)a^2 + (b^2-c^2)a + bc^2 - b^2c \\
 &= (c-b)a^2 - (c^2-b^2)a + bc(c-b) \\
 &= (c-b)a^2 - (c-b)(c+b)a + bc(c-b) \\
 &= (c-b) \{ a^2 - (c+b)a + bc \} \\
 &= (c-b)(a-c)(a-b) \\
 &= (a-b)(b-c)(c-a)
 \end{aligned}$$

$$\begin{aligned}
 97. \quad & (a+b)(b+c)(c+a) + abc \\
 &= (a+b)(a+c)(b+c) + abc \\
 &= \{ a^2 + (b+c)a + bc \} (b+c) + abc \\
 &= (b+c)a^2 + (b+c)^2a + bc(b+c) + abc \\
 &= (b+c)a^2 + (b^2 + 3bc + c^2)a + bc(b+c) \\
 &= \{ a + (b+c) \} \{ (b+c)a + bc \} \\
 &= (a+b+c)(ab+bc+ca)
 \end{aligned}$$

$$\begin{aligned}
 98. \quad & (a+b+c)^3 - a^3 - b^3 - c^3 \\
 &= \{ (a+b+c) - a \} \{ (a+b+c)^2 + a(a+b+c) + a^2 \} \\
 &\quad - (b^3 + c^3) \\
 &= (b+c)(3a^2 + b^2 + c^2 + 3ab + 2bc + 3ca) \\
 &\quad - (b+c)(b^2 - bc + c^2) \\
 &= (b+c)(3a^2 + 3ab + 3bc + 3ca) \\
 &= 3(b+c) \{ a^2 + (b+c)a + bc \} \\
 &= 3(a+b)(b+c)(c+a)
 \end{aligned}$$

$$\begin{aligned}
 99. \quad & a^3(b-c) + b^3(c-a) + c^3(a-b) \\
 &= (b-c)a^3 - (b^3 - c^3)a + bc(b^2 - c^2) \\
 &= (b-c)a^3 - (b-c)(b^2 + bc + c^2)a + bc(b+c)(b-c) \\
 &= (b-c) \{ a^3 - (b^2 + bc + c^2)a + bc(b+c) \} \\
 &= -(b-c) \{ (a-c)b^2 + c(a-c)b - a^3 + ac^2 \} \\
 &= -(b-c) \{ (a-c)b^2 + c(a-c)b - a(a+c)(a-c) \} \\
 &= -(b-c)(a-c) \{ b^2 + cb - a(a+c) \} \\
 &= -(b-c)(a-c)(b-a)(b+a+c) \\
 &= -(a-b)(b-c)(c-a)(a+b+c)
 \end{aligned}$$

$$100. \quad x^3 + y^3 + z^3 - 3xyz$$

$$= (x+y)^3 - 3xy(x+y) + z^3 - 3xyz$$

$$= \{(x+y) + z\} \{(x+y)^2 - z(x+y) + z^2\} - 3xy(x+y+z)$$

$$= (x+y+z)(x^2 + 2xy + y^2 - zx - yz + z^2 - 3xy)$$

$$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$$