

加法定理 part3 [証明(サイン)]

Q 次の問題に答えて、知識をアウトプットしよう！！

問題1

次の三角比の値を求めよ。

(1) $\sin 75^\circ$
 $= \sin(45^\circ + 30^\circ)$ \downarrow 加法定理
 $= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$ $\downarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
 $= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$
 $= \frac{\sqrt{6} + \sqrt{2}}{4}$

(3) $\sin 195^\circ$
 $= \sin(150^\circ + 45^\circ)$ \downarrow 加法定理
 $= \sin 150^\circ \cos 45^\circ + \cos 150^\circ \sin 45^\circ$
 $= \frac{1}{2} \times \frac{1}{\sqrt{2}} + (-\frac{\sqrt{3}}{2}) \times \frac{1}{\sqrt{2}}$
 $= \frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2}$
 $= \frac{\sqrt{2} - \sqrt{6}}{4}$

<別解>
 $\sin 195^\circ$
 $= \sin(180^\circ + 15^\circ)$
 $= -\sin 15^\circ$

(2) $\sin \frac{1}{12}\pi = \frac{1}{12} \times 180^\circ = 15^\circ$
 $= \sin 15^\circ = \sin(45^\circ - 30^\circ)$ \downarrow 加法定理
 $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$
 $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$
 $= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$
 $= \frac{\sqrt{6} - \sqrt{2}}{4}$

(4) $\sin \frac{35}{12}\pi = \frac{35}{12} \times 180^\circ = 35 \times 15^\circ = 525^\circ$ 三角函数の性質
 $= \sin 525^\circ = \sin(360^\circ + 165^\circ) = \sin 165^\circ$
 $= \sin(120^\circ + 45^\circ)$ \downarrow 加法定理
 $= \sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ$
 $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + (-\frac{1}{2}) \times \frac{1}{\sqrt{2}}$
 $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$
 $= \frac{\sqrt{6} - \sqrt{2}}{4}$

問題2

$\sin \theta = -\frac{1}{\sqrt{3}}$, $\tan \theta < 0$ のとき、 $\sin(\theta - \frac{\pi}{6})$ の値を求めよ。

$\cos \theta > 0$ ($\because \tan \theta = \frac{\sin \theta}{\cos \theta}$)
 f: "A3".

$\cos \theta = \sqrt{1 - \sin^2 \theta}$
 $= \sqrt{1 - (-\frac{1}{\sqrt{3}})^2}$
 $= \sqrt{1 - \frac{1}{3}}$
 $= \sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}}$

$\sin(\theta - \frac{\pi}{6})$ \downarrow 加法定理
 $= \sin \theta \cos \frac{\pi}{6} - \cos \theta \sin \frac{\pi}{6}$
 $= -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{\sqrt{3}} \times \frac{1}{2}$
 $= -\frac{1}{2} - \frac{\sqrt{2}}{2\sqrt{3}}$ $\rightarrow \frac{\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2} \times \sqrt{3}}{2 \times 3} = \frac{\sqrt{6}}{6}$
 $= -\frac{1}{2} - \frac{\sqrt{6}}{6} = -\frac{3 + \sqrt{6}}{6}$