

練習問題で in. → out.

加法定理 part2 [証明 2(コサイン)]

1min.

out.



次の問題に答えて、知識をアウトプットしよう！！

問題 1

次の三角比の値を求めよ。

$$(1) \cos 75^\circ$$

$$\begin{aligned} &= \cos(45^\circ + 30^\circ) \quad \text{加法定理} \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \quad 2 \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad) \text{通分} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$(2) \cos \frac{1}{12}\pi$$

$$\frac{1}{12}\pi = \frac{1}{12} \times 180^\circ = 15^\circ \text{ より。}$$

$$\begin{aligned} \cos \frac{1}{12}\pi &= \cos 15^\circ = \cos(45^\circ - 30^\circ) \quad \text{加法定理} \\ &= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \quad 2 \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2} \quad) \text{通分} \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$(3) \cos 195^\circ$$

$$\begin{aligned} &= \cos(150^\circ + 45^\circ) \quad \text{加法定理} \\ &= \cos 150^\circ \cos 45^\circ - \sin 150^\circ \sin 45^\circ \\ &= -\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \\ &= -\frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} \quad) \text{通分} \\ &= -\frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$(4) \cos \frac{35}{12}\pi$$

$$\begin{aligned} \frac{35}{12}\pi &= \frac{35}{12} \times 180^\circ = 35 \times 15^\circ = 525^\circ \\ &= 165^\circ + 360^\circ \text{ より。} \end{aligned}$$

性質

$$\begin{aligned} \cos \frac{35}{12}\pi &= \cos(165^\circ + 360^\circ) = \cos 165^\circ \\ &= \cos(120^\circ + 45^\circ) \quad \text{加法定理} \\ &= \cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ \\ &= -\frac{1}{2} \times \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} \\ &= -\frac{\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

問題 2

$\cos \theta = \frac{1}{\sqrt{3}}$, $\tan \theta < 0$ のとき、 $\cos(\theta - \frac{\pi}{3})$ の値を求めよ。

$$\begin{aligned} \cos \theta > 0, \tan \theta < 0 \text{ より, } \sin \theta < 0 \text{ です。} \\ \sin \theta &= -\sqrt{1 - \cos^2 \theta} \quad (\text{相互関係}) \\ &= -\sqrt{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= -\sqrt{1 - \frac{1}{3}} \\ &= -\sqrt{\frac{2}{3}} = -\frac{\sqrt{2}}{\sqrt{3}} \end{aligned}$$

より 60°

$$\begin{aligned} \cos\left(\theta - \frac{\pi}{3}\right) &= \cos \theta \cos \frac{\pi}{3} + \sin \theta \sin \frac{\pi}{3} \\ &= \frac{1}{\sqrt{3}} \times \frac{1}{2} + \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \\ &= \frac{1}{2\sqrt{3}} + \frac{\sqrt{2}}{2} = \\ &\stackrel{\text{分母有理化}}{=} \frac{\sqrt{3} + 3\sqrt{2}}{6} \end{aligned}$$